How to obtain an overall reduction of individual income deprivations by means of a finite sequence of T 2 -transformations? A solution and a well-suited algorithm

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How to obtain an overall reduction of individual income deprivations by means of a finite sequence of T_2-transformations? A solution and a well-suited algorithm

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Abstract

Finite sequences of T_2-*transformations* is one of the transfer mechanisms that can be used to achieve the reduction of income inequality such that no individual will experiment an increase of his total deprivation: some individuals may notice a reduction while others may record no change. We define the conditions for which a finite sequence of T_2-*transformations* will guarantee that every individual experiments a reduction in his total deprivation and we provide a suited algorithm for such a possibility.

Keywords: Inequality, Income, Deprivation, T_2 transformation, Algorithm.

1. Introduction

Income inequality refers to the extent to which incomes are distributed unequally among a population.[1] Progressive transfers (see Dalton (1920)) are the most popular mechanisms often used for the reduction of income inequality. A progressive transfer induces an income transfer from richer to poorer individuals. Nonetheless, while helping in reducing income inequality between two people, it is known that one of the main drawbacks of progressive transfers is that they can increase the income differences between each of these two people and the others. This led some authors to reject the principle of progressive transfers in a search of inequality reduction (see for instance, Amiel and Cowell (1992, 1999). Income differences can also allow to define the *individual's deprivation* as people compare themselves with some reference individual or group of individual within the society rather than with the whole society. See for instance, the papers by Chakravarty et al. (1995), Kakwani (1984), Runciman (1966), Silber (1999) and Yitzhaki (1979, 1983).

According to Chateauneuf and Moyes (2005)[2], a way to reduce income inequality while acting on individual deprivations is to introduce a notion of solidarity among the individuals concerned by the transfer process. They suggested some mechanisms (transformations) of which progressive transfers are particular cases. The successive applications of these transformations will result in a distributional improvement according to the individual deprivations. Among the suggested transformations, we are concerned with the T_2-*transformations* which stipulates that if some income is taken from a rich individual, then the same amount has to be taken from every non poorer individual and it is no longer necessary that individuals poorer than the transfer recipient benefit also from some equal additional income. When

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[1]The *Gini index* (Gini, 1912), widely used in various fields (economics, finance, engineering, etc.), is the most popular tool for income inequality measurement. This index is often used for comparison of income distributions across different groups of people or groups of states (countries). In such a framework, the index varies between 0 and 1: a value of 0 indicates a situation of complete equality under which everyone gets the same income; conversely, a value of 1 corresponds to a situation of perfect inequality in which one person has all the income and the others have nothing.

[2]See also Moyes (2007) and the related paper.

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a less unequal income distribution x can be obtained from an income distribution y by means of a finite sequence of T_2-transformations, no individual will experiment an increase of his total deprivation, some individuals may notice a reduction while others may record no change. Is it not possible to do better? To do better would be to (always) ensure a situation in which by means of T_2-transformations, each individual experiments a reduction in his total deprivation. That is our objective in this paper. So, we define the conditions for which a finite sequence of a T_2-*transformation* will guarantee that every individual experiments a reduction in his total deprivation and we provide an algorithm for such a possibility.

The rest of the paper is organized as follows. In Section 2, we set the framework with some basic notations then we formally define the problem under consideration in this paper. Our main results are provided in Section 3 where we also suggest a well-suited algorithm for solving our problem. Section 4 concludes.

2. Basic definitions and the considered problem

2.1. Basic definitions

Let U denotes the set of income distributions z in $\mathbb{R}^d$ such that $z = (z_1, z_2, \ldots, z_{d-1}, z_d)$ in $\mathbb{R}^d$ (with $d \in \mathbb{N}^*$), $0 \leq z_i \leq z_j$ for any $1 \leq i \leq j \leq d$ and $\sum_{i=1}^{d} z_i = S$ (for some fixed positive value S). In the distribution z, the incomes are sorted by ascending order; so z_1 designates the income of the poorest individual and z_d the income of the richest individual.

We introduce the linear mapping $D : \mathbb{R}^d \to \mathbb{R}^{d-1}$ defined for z in $\mathbb{R}^d$ by

$$D(z) = (D_1(z), D_2(z), ..., D_{d-1}(z))$$

where components are given for $i = 1, .., d-1$ by

$$D_i(z) = (d-i)z_i - \sum_{j=i+1}^{d} z_j$$

The vector $D(z)$ is the distribution of the individual total income deprivations $D_i(z)$ which is, for each individual i, the sum of income pairwise differences between z_i and the z_j such that $z_j > z_i$.

Given two income distribution x and y in $\mathcal{U}_n$, x is obtained from y by means of a progressive transfer, if there exists a positive value Δ ($\Delta > 0$) and two individual i and j such that

- $x_g = y_g$ for all $g \neq i, j$,
- $x_i = y_i + \Delta$ and $x_j = y_j - \Delta$ with $\Delta \leq \frac{y_j - y_i}{2}$.

In principle, a progressive transfer does not reverse the relative positions of the individuals (the donor(s)) who contribute to the transfer and those (the recipient(s)) who benefit from it. As in Chateauneuf and Moyes (2005), we assume that the progressive transfer is rank-preserving in the sense that the relative positions of all the individuals are unaffected; this is tantamount to impose the following condition to a progressive transfer:

$$\forall p \neq q, (x_q - x_p)(y_q - y_p) \geq 0 \tag{1}$$

A vector x in $\mathbb{R}^d$ is obtained from y in $\mathbb{R}^d$ by means of a T_2-*transformation*, denoted by $x = \Gamma_{h,k}^{\epsilon}(y)$, if there exist a positive value ϵ and two indexes $h \in \{1, .., d-1\}$ and $k \in \{1, .., d\}$ with $k \geq h+1$ such that Equation (1) holds and

- $x_i = y_i$ for $i \in \{1, .., h-1\} \cup \{h+1, .., k-1\}$,
- $x_h = y_h + \Delta$ with $\Delta = (d-k+1)\epsilon$,
- $x_i = y_i - \epsilon$ for $i = k, .., d$,

Let us use an example to illustrate how a T_2-*transformation* operates. This example will also motivate the concern of this paper.

Example 1. Assume $d = 5$ and consider the following income distribution: $y = (3, 4, 5, 6, 7)$. The reader can easily check what follows:

$$\begin{aligned} u = (3,4,6,6,6) &= \Gamma^1_{3,5}(y) \\ v = (3,5,5,6,6) &= \Gamma^1_{2,5}(y) \\ w = (4,4,5,6,6) &= \Gamma^1_{1,5}(y) \end{aligned}$$

By computing the corresponding vectors $D(.)$, we get:

Distributions	$D(.)$
y	$(-10, -6, -3, -1)$
u	$(-10, -6, 0, 0)$
v	$(-10, -2, -2, 0)$
w	$(-5, -5, -2, 0)$

It is known that when a distribution x is obtained from a distribution y by means of a T_2-transformation, x is always less unequal than y. More, while going from y to x, the guarantee is that no individual will experiment an increase of his total deprivation. In some cases, some individuals will not experience any change in their total deprivation while others will notice a reduction. What we get in Example 1 gives an illustration of that. In this paper, we want to focus on how to reach a situation in which each individual records a reduction in his total deprivation while obtaining a distribution x from a distribution y by means of T_2-transformations. This defines the main problem we deal with. Let us formally define our problem.

2.2. The considered problem

Given two vectors x and y in U, we wish to show that x can be obtained from y by means of a finite sequence (z^n) of T_2*-transformations* in U such that

$$D(x - y) = D(x) - D(y) > 0,$$

in the sense that:

- $z^0 = y$,
- $(z^n) \subset U$,
- z^{n+1} is obtained from z^n by a T_2*-transformation*,
- $(z^n) \to x$ finitely.

Let (z^n) be a sequence in U such that $z^0 = y$ and $z^{n+1} = \Gamma^{\epsilon_n}_{h_n,k_n}(z^n)$ where h_n and k_n are indexes such that $h_n \in \{1, .., d-1\}$, $k_n \in \{1, .., d\}$ and $k_n \geq h_n + 1$ for all $n \geq 0$. For the sake of simplicity, we set $D^n = D(\delta^n)$ with $\delta^n = x - z^n$.

3. Main results

As we going to see, the solution to our problem is to establish sufficient conditions so that the goal is achieved.

3.1. Sufficient conditions

Propositions 1 and 2 describe how to set the parameters of the transformation in order to ensure that $z^{n+1} = \Gamma^{\epsilon_n}_{h_n,k_n}(z^n)$ belongs to U. These propositions define the sufficient conditions for $z^n \in U$ and $D(z^n) \leq D(z^{n+1}$.

Proposition 1. *For any $n \geq 0$, we have $D(z^{n+1}) \geq D(z^n)$.*

Proposition 2. *Suppose that $z^n \in U$ for some $n \geq 0$. Then the next iterate z^{n+1} belongs to U provided that each of the following conditions are satisfied:*

$$\epsilon_n \leq \frac{z^n_{h_n+1} - z^n_{h_n}}{d - k_n + 1} \quad (\textit{if } k_n > h_n + 1), \tag{2}$$

$$\epsilon_n \leq z^n_{k_n} - z^n_{k_n-1} \quad (\textit{if } k_n > h_n + 1), \tag{3}$$

$$\epsilon_n \leq \frac{z^n_{h_n+1} - z^n_{h_n}}{d - h_n + 1} \quad (\textit{if } k_n = h_n + 1). \tag{4}$$

Proof. It is easy to see that $\sum_{i=1}^d z_i^{n+1} = \sum_{i=1}^d z_i^n = S$, so it remains to ensure that $z_i^{n+1} \leq z_j^{n+1}$ for $1 \leq i \leq j \leq d$. Clearly by the definition of the T_2-*transformation*, it suffices to ensure that $z^{n+1}_{h_n} \leq z^{n+1}_{h_n+1}$ and $z^{n+1}_{k_n-1} \leq z^{n+1}_{k_n}$. We recall that

$$\begin{aligned} z^{n+1}_{h_n} &= z^n_{h_n} + \Delta_n, \text{ where } \Delta_n = (d - k_n + 1)\epsilon_n \\ z^{n+1}_{k_n} &= z^n_{k_n} - \epsilon_n. \end{aligned}$$

Suppose that $k_n > h_n + 1$. Then we have $z^{n+1}_{h_n+1} = z^n_{h_n+1}$, so that the condition $z^{n+1}_{h_n} \leq z^{n+1}_{h_n+1}$ is equivalent to $z^n_{h_n} + \Delta_n \leq z^n_{h_n+1}$, namely (2). Moreover, we have $z^{n+1}_{k_n-1} = z^n_{k_n-1}$, so that the condition $z^{n+1}_{k_n-1} \leq z^{n+1}_{k_n}$ becomes $z^n_{k_n-1} \leq z^n_{k_n} - \epsilon_n$, that is (3). In the case when $k_n = h_n + 1$ we have $z^{n+1}_{h_n+1} = z^n_{h_n+1} - \epsilon_n$, so that the condition $z^{n+1}_{h_n} \leq z^{n+1}_{h_n+1}$ (which also reduces to $z^{n+1}_{k_n-1} \leq z^{n+1}_{k_n}$) becomes $z^n_{h_n} + \Delta_n \leq z^n_{h_n+1} - \epsilon_n$, that is (4). □

First, let us highlight some important relationships that will be useful in the sequel.

Remark 1. *Let us set $G^n = D(z^{n+1} - z^n)$. It is easily checked that*

- $G_i^n = 0$ *for* $i = 1, ..., h_n - 1$
- $G^n_{h_n} = (d - h_n + 1)\Delta_n$
- $G_i^n = \Delta_n$ *for* $i = h_n + 1, ..., k_n - 1$ *(if* $k_n > h_n + 1$*)*
- $G_i^n = 0$ *for* $i = k_n, ..., d - 1$.

Proposition 3 provides the sufficient conditions for $D(z^n) \leq D(x)$.

Proposition 3. *Suppose that $D^n \geq 0$ and the following conditions are reached:*

- $(d - h_n + 1)\Delta_n \leq D^n_{h_n}$, *or equivalently*

$$\epsilon_n \leq \frac{D^n_{h_n}}{(d - h_n + 1)(d - k_n + 1)},$$

- $\Delta_n \leq D_i^n$ *for* $i = h_n + 1, ..., k_n - 1$ *(if* $k_n > h_n + 1$*), or equivalently*

$$\epsilon_n \leq \frac{D_i^n}{(d - k_n + 1)},$$

Then it holds that $D(z^{n+1}) \leq D(x)$.

Proof. Clearly, by linearity of $D(.)$, we have $D(x - z^{n+1}) = D(x - z^n) - D(z^{n+1} - z^n)$, that is $D(x - z^{n+1}) = D^n - G^n$. As a consequence, $D(x - z^{n+1}) \geq 0$ is equivalent to $G^n \leq D^n$, which obviously leads to the desired result. □

As one can notice, the relations defined in Propositions 2 and 3 essentially concern how the choice of parameters should be made; a choice that is crucial given the objective pursued. Equipped with these relation, we are now in position to construct an algorithm for solving our problem in concern here.

3.2. *Construction of a well-suited algorithm*

Prior to propose an algorithm for solving our problem, we first need to state some results that open the way to the implementation of this algorithm. Propositions 4 to 6 describe preliminaries for Propositions 7 to 8 which assures us the solution to our problem, that by a T_2-transformation, each individual benefits from a reduction of his total deprivation.

Proposition 4. *For $i = 1, ..., d-2$, the following estimates are reached:*
e1) $D^n_{i+1} - D^n_i = (d-i)(\delta^n_{i+1} - \delta^n_i)$*;*

e2) $D^n_{i+1} - D^n_i = (d-i)(x^n_{i+1} - x^n_i) + (d-i)(z^n_i - z^n_{i+1})$*;*

e3) $D^n_{i+1} - D^n_i \geq (d-i)(z^n_i - z^n_{i+1})$*;*

Proof. Using the definition of D^n_i, we have

$$\begin{aligned} D^n_{i+1} - D^n_i &= (d-i-1)\delta^n_{i+1} - \sum_{j=i+2}^{d} \delta^n_j - (d-i)\delta^n_i + \sum_{j=i+1}^{d} \delta^n_j \\ &= (d-i-1)\delta^n_{i+1} - (d-i)\delta^n_i + \delta^n_{i+1} \\ &= (d-i)(\delta^n_{i+1} - \delta^n_i), \end{aligned}$$

that is (e1). From the definition of δ^n_i, we also have

$$\delta^n_{i+1} - \delta^n_i = (x^n_{i+1} - z^n_{i+1}) - (x^n_i - z^n_i) = (x^n_{i+1} - x^n_i) + (z^n_i - z^n_{i+1}),$$

which by (e1) leads to (e2). Moreover observing that $x^n_{i+1} - x^n_i \geq 0$ (as $x \in U$), we get

$$\delta^n_{i+1} - \delta^n_i \geq z^n_i - z^n_{i+1}, \tag{5}$$

which by (e1) leads to (e3). □

Proposition 5. *For $i = 1, ..., d-1$, $D^n_i \leq (d-i)(z^n_d - z^n_i) - (d-i)(x_{i+1} - x_i)$.*

Proof. We recall that $D^n_i = (d-i)(x_i - z^n_i) - \sum_{j=i+1}^{d}(x_j - z^n_j)$. Then, by $x_j \leq x_{j+1}$ (hence $x_j \geq x_{i+1}$ for $j \geq i+1$), we obtain

$$D^n_i \leq (d-i)(x_i - z^n_i) - (d-i)x_{i+1} + \sum_{j=i+1}^{d} z^n_j.$$

Hence, from $z^n_j \leq z^n_{j+1}$ (hence $z^n_j \geq z^n_d$ for $j \geq 0$), we get

$$D^n_i \leq (d-i)(x_i - z^n_i) - (d-i)x_{i+1} + (d-i)z^n_d,$$

or equivalently

$$D^n_i \leq (d-i)(z^n_d - z^n_i) - (d-i)(x_{i+1} - x_i),$$

that is the desired result. □

Proposition 6. *The following properties are satisfied:*

p1) For $i = 1, .., d-2$, $z^n_{i+1} = z^n_i \Longrightarrow D^n_{i+1} \geq D^n_i$;

p2) For $i = 1, .., d-1$, $D^n_i > 0 \Longrightarrow z^n_i < z^n_d$.

Proof. (p1) is obviously deduced from Proposition 4. (p2) is obtained from Proposition 5 by the fact that $x_{i+1} - x_i \geq 0$ together with $D^n_i > 0$ □

Proposition 7. *Suppose that $D^n > 0$ and set $j_n = \max\{i \in \{1,..,d-1\} \mid D^n_i > 0\}$. Then it holds that $D^n_{j_n} > 0$ and $z^n_{j_n+1} > z^n_{j_n}$.*

Proof. Clearly, we have either $z^n_{j_n+1} > z^n_{j_n}$ or $z^n_{j_n+1} = z^n_{j_n}$. The latter situation in light of Proposition 6 ensures that $D^n_{j_n+1} \geq D^n_{j_n}$ (hence $D^n_{j_n+1} > 0$) when $j_n \leq d-2$, which is absurd. It is also observed from Proposition 6 that $z^n_d > z^n_{d-1}$ whenever $D^n_{d-1} > 0$. This leads immediately to the desired result. □

Remark 2. *Recall that $G^n = D(z^{n+1} - z^n)$. It is then immediate that $G_n = D(x - z^n) - D(x - z^{n+1})$, or equivalently $D^{n+1} - D^n = -G^n$. This observation together with Remark 1 amounts to*

- $D^{n+1}_i = D^n_i$ *for* $i = 1,..,h_n - 1$
- $D^{n+1}_{h_n} = D^n_{h_n} - (d - h_n + 1)\Delta_n = D^n_{h_n} - (d - h_n + 1)(d - k_n + 1)\epsilon_n$
- $D^{n+1}_i = D^n_i - \Delta_n$ *for* $i = h_n + 1, ..., k_n - 1$ *(if* $k_n > h_n + 1$*)*
- $D^{n+1}_i = D^n_i$ *for* $i = k_n, ..., d - 1$.

Proposition 8. *Set $\alpha^n_j = z^n_{j+1} - z^n_j$ and $\beta^n_j = \dfrac{D^n_j}{d-j}$. Then the following results hold:*

(r1) $\beta^n_{d-1} \leq \alpha^n_{d-1}$;

(r2) For $j = 1, ..., d-2$, $\alpha^n_j < \beta^n_j \Longrightarrow D^n_{j+1} > 0$;

Proof. Let us prove (r1). Clearly, we have $\alpha^n_{d-1} = z^n_d - z^n_{d-1}$, while it is a simple matter to see that

$$\begin{aligned}
\beta^n_{d-1} = D^n_{d-1} &= (x_{d-1} - z^n_{d-1}) - \sum_{i=d}^{d}(x_i - z^n_i) \\
&= (x_{d-1} - z^n_{d-1}) - (x_d - z^n_d) \\
&= (z^n_d - z^n_{d-1}) - (x_d - x_{d-1}).
\end{aligned}$$

As a consequence, we have $\beta^n_{d-1} = \alpha^n_{d-1} - (x_d - x_{d-1})$, so that $\beta^n_{d-1} \leq \alpha^n_{d-1}$ (due to $x_d \geq x_{d-1}$ as $x \in U$).
Now we prove (r2). Suppose that $\alpha^n_j < \beta^n_j$ for some $j = 1, ..., d-2$, hence, we equivalently have

$$D^n_j - (d-j)(z^n_{j+1} - z^n_j) > 0. \tag{6}$$

Moreover, using the definition of D^n_j we have

$$\begin{aligned}
&D^n_j - (d-j)(z^n_{j+1} - z^n_j) \\
&\qquad = (d-j)(x_j - z^n_j) - \textstyle\sum_{i=j+1}^{d}(x_i - z^n_i) - (d-j)(z^n_{j+1} - z^n_j) \\
&\qquad = (d-j)(x_j - z^n_{j+1}) - \textstyle\sum_{i=j+1}^{d}(x_i - z^n_i) \\
&\qquad = (d-j)(x_j - x_{j+1}) + (d-j)(x_{j+1} - z^n_{j+1}) - (x_{j+1} - z^n_{j+1}) - \textstyle\sum_{i=j+2}^{d}(x_i - z^n_i) \\
&\qquad = (d-j)(x_j - x_{j+1}) + (d-j-1)(x_{j+1} - z^n_{j+1}) - \textstyle\sum_{i=j+2}^{d}(x_i - z^n_i)
\end{aligned}$$

It follows that $D^n_j - (d-j)(z^n_{j+1} - z^n_j) = (d-j)(x_j - x_{j+1}) + D^n_{j+1}$. Consequently, inequality (6) can be rewritten as $D^n_{j+1} > (d-j)(x_{j+1} - x_j)$; hence, recalling that $x_{j+1} - x_j \geq 0$ (as $x \in U$), we deduce that $D^n_{j+1} > 0$, that is the desired result. □

Remark 3. *Suppose for some $n \geq 0$ that $z^{n+1} = \Gamma^{\epsilon_n}_{h_n,k_n}(z^n)$ with $k_n = h_n + 1$ (for some $\epsilon_n \geq 0$ and $h_n \in \{1,..,d-1\}$), where z^n satisfied*

$$z^n \in U \text{ and } D(y) \leq D(z^n) \leq D(x). \tag{7}$$

According to Propositions 1, 2 and 3, we then have

$$z^{n+1} \in U \text{ and } D(y) \leq D(z^{n+1}) \leq D(x), \tag{8}$$

provided that

$$\epsilon_n \leq \min\{\frac{\beta^n_{h_n}}{d - h_n + 1}, \quad \frac{\alpha^n_{h_n}}{d - h_h + 1}\},$$

where $\alpha^n_j = z^n_{j+1} - z^n_j$ *and* $\beta^n_j = \dfrac{D^n_j}{d - j}$.

We can now introduce an algorithm for solving our problem defined in Section 2.

Algorithm 1 The considered Algorithm

Initialization: Set $z^0 = y$ and $n = 0$;
Step 1: set h_n as the largest value $i \in \{1, .., d-1\}$ such that $D_i(x - z^n) > 0$;
Step 2: set $k_n = h_n + 1$;
Step 3: set $\epsilon_n = \frac{D^n_{h_n}}{(d-h_n)(d-h_n+1)}$;
Step 4: Compute $z^{n+1} = \Gamma^{\epsilon_n}_{h_n,k_n}(z^n)$ (which ensures that $D_{h_n}(x - z^{n+1}) = 0$);
Step 5: If $z^{n+1} \neq x$ then $n \leftarrow n + 1$ and go to Step 1; otherwise stop.

Theorem 1. *The sequence* (z^n)*, generated by the considered Algorithm 1, is included in U and converges finitely to x.*

Proof. Recall that condition (7) is satisfied at the first iteration $n = 0$. To get the result, we prove that (at each iteration n) condition (7) implies that (8) holds and we face one of the following situation:
(A_n) $z^n = x$;
(B_n) $h_{n+1} \leq h_n - 1$.

To that end we consider the values $\alpha^n_j = z^n_{j+1} - z^n_j$ and $\beta^n_j = \dfrac{D^n_j}{d - j}$ introduced in Prop. 8 and Remark 3. Suppose that $z^n \neq x$ and the condition (7) holds at some iteration n. It is clear that h_n is well-defined and we additionally have $x_{h_n+1} > x_{h_n}$ and $D^n_{h_n} > 0$ (thanks to Prop. 7). We also have $\beta^n_{h_n} \leq \alpha^n_{h_n}$ (from Prop. 8), so that the choice of ϵ_n in the algorithm satisfies $\epsilon_n = \min\{\frac{\beta^n_{h_n}}{d-h_n+1}, \quad \frac{\alpha^n_{h_n}}{d-h_n+1}\}$. It follows that (8) holds (from Remark 3) and we immediately deduce that $D^{n+1}_{h_n} = 0$. It is then easily checked that $D^{n+1}_j = 0$ for $j = h_n, ..., d-1$, because $0 \leq D^{n+1}_j \leq D^n_j$ for $j = 1, ..., d-1$ (from Remarks 2 and 3), so that $h_{n+1} \leq h_n - 1$. □

4. Conclusion

In this paper, we have defined the conditions for which a finite sequence of T_2-*transformations* will always guarantee that when moving to a less unequal income distribution, every individual experiments a reduction in his total deprivation and we have provided an algorithm for such a possibility.

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